5 Geometry

In this chapter you will answer…

- What is the sum of the angles in a triangle?
- How many sides are there in a regular heptagon?
- What is the name of the line from the centre of a circle to the edge?
- How many millimetres are there in a metre?
- What is the formula for the volume of a cone?
- How many lines of symmetry are there in a square?

5.1 Angles

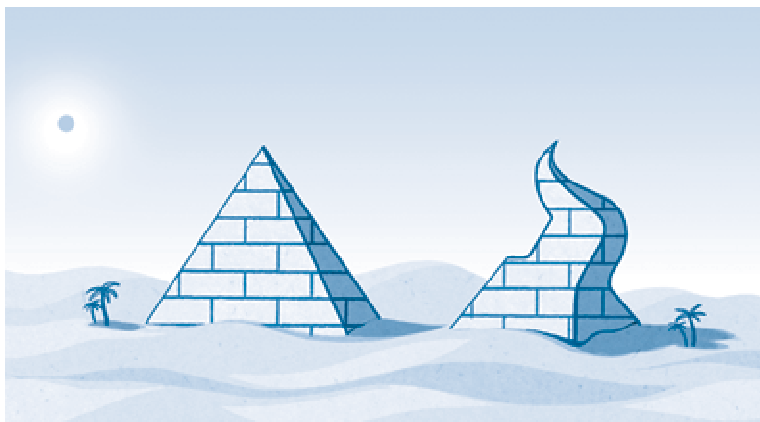
Angles are used to measure turning and rotation.

Angles are measured in degrees. The symbol for a degree is a small circle to the right of the number.

Example

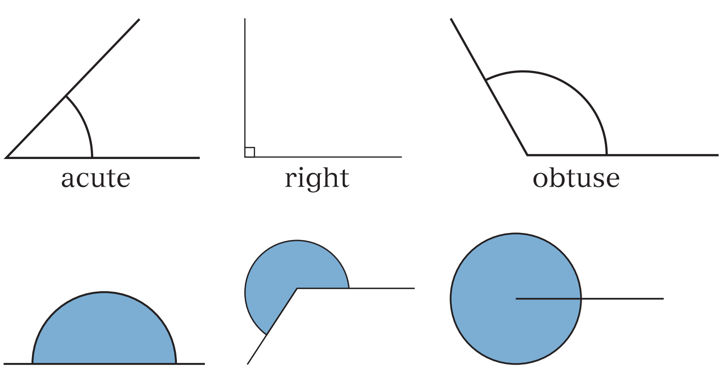
You write fifty degrees as 50°.

- An acute angle is less than 90°.
- A right angle is exactly 90°.
- An obtuse angle is less than 180° but more than 90°.
- A straight angle is exactly 180°.
- A reflex angle is less than 360° but more than 180°.
- A full rotation is 360°.



Geometry can be useful!

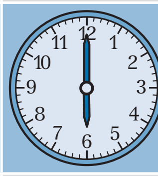
Be Careful. When teachers talk about geometry they use many words that may be familiar, or seem easy to find in a dictionary. But these words have exact mathematical meanings and you need to make sure that you understand the 'maths word', not just the English word.



Try to find out why 360° is used for a full circle.

Exercise

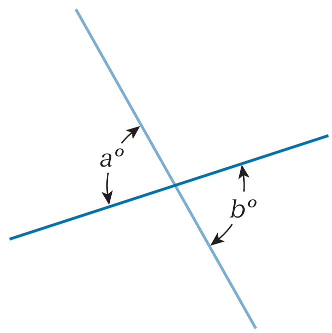
1 Use the diagrams to complete this table:

	Clock face	Name of angle	Size of angle
		Right angle	90° 270°
a			240°
b			
c			225°
d			
e			

5.2 Angles between straight lines

When two straight lines cross, angles are made that have special names and connections.

Vertically opposite angles are equal, so $a° = b°$

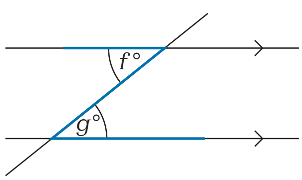


5.3 Angles in parallel lines

When a straight line crosses parallel lines, the angles also have special properties.

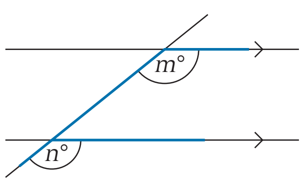
1 **Alternate** angles are equal, so $f° = g°$

Alternate angles make a **Z** shape



2 **Corresponding** angles are equal, so $m° = n°$

Corresponding angles make an **F** shape



KEY WORD

Parallel Lines are parallel if they are always the same distance apart and will never meet. E.g. Railway lines are parallel

Exercise

2 **Look at these letters carefully. Then choose a word(s) from the box to complete each sentence.**

acute obtuse right-angles parallel 360° reflex full turn 180°

The first one has been done for you.

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a **M** has three <u>acute</u> angles.
b **T** has two ……………. angles.
c **M**, **H** and **E** have ……………. lines.
d **A** has two ……………. angles.
e The ……………. angles in **E** add up to ……………. degrees.
f **I** is a straight line with an angle of ……………. degrees.
g **V** has one ……………. angle and one ……………. angle.
h **O** contains a ……… …….. with an angle of …………….

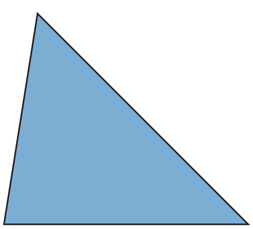
5.4 Triangles

A triangle is the simplest type of polygon. It has three straight sides and three angles.

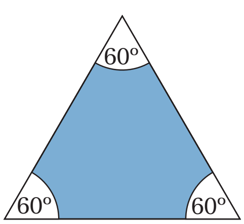
The angles in a triangle add up to 180°.

There are different names for particular triangles:

In a **scalene** triangle, all the angles are different sizes and all the sides are different lengths.



In an **equilateral** triangle, all the sides are the same length and all the angles are the same size

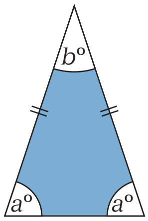


KEY WORD

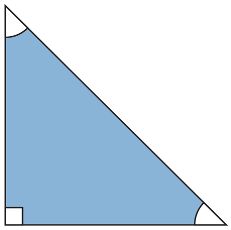
Polygon a shape with three or more straight sides

Why does a polygon have to have at least three sides? Can you draw a polygon with two straight sides?

In an **isosceles** triangle, two angles are the same size and two sides are the same length



In a **right-angled** triangle, there is one angle of 90°.



The Greeks and Romans thought that Geometry was the most important science.

The Greek and Roman languages have given the different angles and triangles their special names.

An **acute** angle is sharp and the Latin adjective for 'sharp' is 'acutus'.

A **reflex** angle bends backwards and the Latin verb for 'bend' is 'flectere'.

A **scalene** triangle is not regular and the Greek adjective for 'uneven' is 'scalenos'.

An **isosceles** triangle has two sides that are the same length, and the Greek words for 'equal' and 'legs' are 'iso' and 'sceli'.

Exercise

3 Find the name of each triangle:

a "I am the most elegant triangle, as two of my sides are always the same."
I am a/an …………… triangle.

b "I am the most reliable triangle, as I am always the same shape, even when I am a different size."
I am a/an ………… triangle.

c "I am the most interesting triangle as I am always different."
I am a/an …………… triangle.

d "I am the most upright triangle, as I always stand straight".
I am a/an …………… triangle.

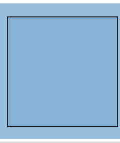
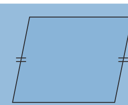
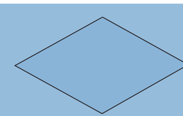
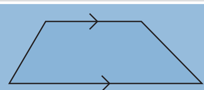
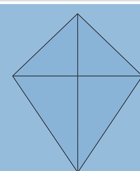
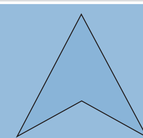
e "I am very special, as I am both upright and elegant."
I am a/an ………… ………… triangle.

5.5 Quadrilaterals

A quadrilateral is a polygon with four straight sides and four angles.

The angles of a quadrilateral add up to 360°.

There are eight different types of quadrilateral and each one has its own name and properties:

			
Square All sides are the same length. All angles are the same size (90°). Opposite sides are parallel.	**Rectangle** Opposite sides are the same length. All angles are the same size (90°). Opposite sides are parallel.	**Parallelogram** Opposite sides are the same length and parallel. Opposite angles are the same size.	**Rhombus** All sides are the same length. Opposite sides are parallel. Opposite angles are the same size.
			
Isosceles trapezium One pair of parallel sides. Base angles are the same size.	**Trapezium** One pair of parallel sides.	**Kite** Two pairs of sides are the same length. One pair of equal angles.	**Arrowhead** Two pairs of sides are the same length. One reflex angle. One pair of equal angles.

Arrowheads are special as they are **concave**.
All the other quadrilaterals are **convex**.

5.6 Polygons with more than four sides

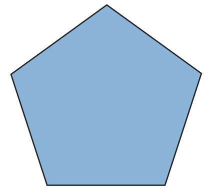
'Polygon' is the name for any shape that is drawn with straight sides.

A **regular** polygon has equal sides and equal angles.

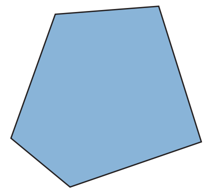
In Geometry 'equal' means the same size or the same length.
The names of the polygons come from Greek or Latin words:

> **KEY WORD**
>
> **Regular** a polygon is regular if all its angles are the same size and all its sides are the same length

Number of sides of the polygon	Number	Latin name of the number	Greek name of the number	Name of Polygon
3	3	Tres	Tria	Triangle
4	4	Quattuor		Quadrilateral
5	5		Pente	Pentagon
6	6		Hexi	Hexagon
7	7		Hepta	Heptagon
8	8	Octo	Octo	Octagon
9	9	Novem		Nonagon
10	10	Decem	Deca	Decagon
12	12	Duodecem	Dodeca	Dodecagon
20	20		Ikosi	Icosahedron



Regular pentagon



Irregular pentagon

Exercise

4 **Wordsearch.** Find the names of the angles and polygons:

S	C	A	L	E	N	E	T	N	Y	E	H
N	O	G	A	C	E	D	O	D	Q	L	E
X	S	V	T	J	O	G	W	U	F	G	X
S	C	E	K	R	A	B	I	N	N	N	A
R	U	I	L	T	I	L	T	O	X	A	G
E	Q	B	P	E	A	A	N	U	A	T	O
F	T	E	M	T	C	A	N	C	S	C	N
L	H	I	E	O	G	S	U	G	W	E	I
E	X	R	K	O	H	T	O	X	L	R	S
X	A	Z	N	Q	E	R	W	S	N	E	Y
L	P	E	N	T	A	G	O	N	I	Y	U
N	O	G	A	T	C	O	W	O	R	R	A

acute	**arrow**	**dodecagon**	**equilateral**	**heptagon**
hexagon	**isosceles**	**kite**	**nonagon**	**obtuse**
octagon	**pentagon**	**rectangle**	**reflex**	**rhombus**
scalene	**triangle**			

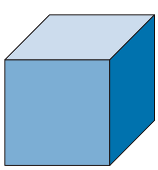
5.7 Solids: shapes in three dimensions

All solids have

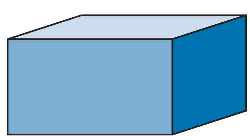
- **faces**, the flat surfaces
- **edges**, where the faces meet
- **vertices (plural of vertex)**, the points where the edges meet.

A solid is a shape formed in three dimensions.

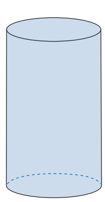
1 A solid with six square faces is called a **cube**.



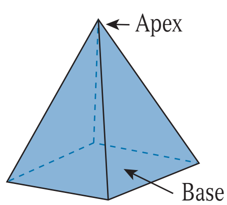
2 A **cuboid** has six faces which are rectangles or squares.



3 The base and top of a **cylinder** are circles.



4 The base of a **pyramid** may be a square, or a rectangle.



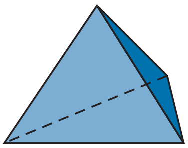
KEY WORDS

Volume the amount of 3-dimensional space occupied by an object

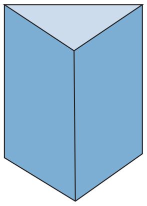
Capacity the amount of space that something contains

Be careful! A shape has volume (the amount of space that it takes up) and capacity, which is the amount that it contains.

5 A pyramid with a triangular base is called a tetrahedron.



6 A triangular prism has a triangle as its cross section. The cross section is the same for the whole length.



A polyhedron is a solid with flat sides. A cylinder is not a polyhedron.

Exercise

5 **Complete this table, and work out the number in the last column for each shape.**

	Shape of faces	Number of faces (F)	Number of edges (E)	Number of vertices (V)	F + V − E
a Cube	Square				
b Cuboid			12		
c Square-based pyramid	One square Four triangles				
d Triangular prism		5			
e Tetrahedron				4	

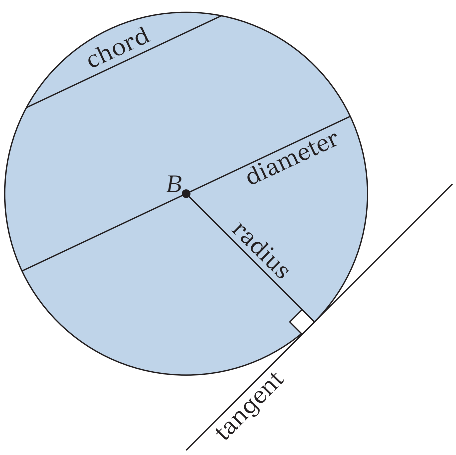
Euler's Formula

The value of F + V − E for any straight-sided solid is called 'Euler's formula'.

In Exercise **5**, did you get the same value each time?

Leonhard Euler, (1707–83) was a Swiss mathematician who made a lasting contribution to the development of mathematics.

5.8 Circles



The **radius** of a circle is the distance from the centre of the circle to the **circumference**.

The **circumference** of a circle is the perimeter, or outside edge.

The **diameter** of a circle is a straight line through the centre from one edge to the opposite edge.

A **chord** is a straight line joining two points on the circumference.

A **tangent** touches the **circumference** and makes a **right angle** with the **radius** at the point of contact.

Important formulae:

Circumference = $\pi \times$ diameter or Circumference = $2 \times \pi \times$ radius

$C = \pi d$ or $C = 2\pi r$

π is a special number.

Remember:
In algebra, when letters are joined up this means multiply.

Facts about π (pi)

- π is the number found when the circumference of a circle is divided by the length of its diameter.
- A circle with a diameter of one metre will have a circumference of 3.14 (3 d.p.) metres.
- This idea has been used for at least 4,000 years.
- It has been known from early times that the value of π was a little bit more than 3.
- The first person to use the symbol π was William Jones in 1706. He used the Greek letter for *P* to represent perimeter.
- π-day is celebrated each year on March 14th.
- Using a computer π has been calculated to 51 billion places.
- π is an irrational number. It cannot be written as a fraction and no pattern has been found for the order of the numbers. The first 10 figures for π are 3.141592654…

Why March 14th?

Talking points

Knowledge of geometry teaches architects to design attractive building.

Work with partner and find five examples of famous buildings that you both like. Try and identify as many of the shapes and angles as you can.

5.9 Metric measures

People measure things every day. It is also an important idea in mathematics.

Metric measures are the standard international measures for length, mass and volume.

The standard units measure mass, not weight. Do you know why?

These are the names of the standard units:

Length		Mass		Volume	
Millimetre	(mm)	Milligram	(mg)	Millilitre	(ml)
Centimetre	(cm)			Centilitre	(cl)
Metre	(m)	Gram	(g)		
Kilometre	(km)	Kilogram	(kg)	Litre	(l)
		Tonne	(t)		

Milli- and kilo-

Many units of measurement begin with milli- or kilo-

'Mille' means one thousand in Latin	A **milli**metre is $\frac{1}{1000}$ of a metre
'Khilloi' means one thousand in Greek	A **kilo**gram is 1000 grams, a **kilo**metre is 1000 metres
'Centum' means one hundred in Latin	A **centi**metre is $\frac{1}{100}$ of a metre

Exercise

6 Which unit would you use to measure these? Choose from the list of units in the box.

kilometres grams litres centimetres millilitres millimetres kilograms

	Unit
a The distance between Athens and Rome	
b The volume of liquid in a can of cola	
c The mass of a shark	
d The length of a fishing rod	
e The mass of a shoe	**grams**
f The volume of water in a pond	
g The width of a leaf	

Exercise

7 Complete these sentences with the correct units.

a The distance to the bus stop is 700 ……………..

b The mass of a newborn baby is 3.2 …………..

c The length of a pencil is 13 …………

d To cook rice measure 800………. of water for 250 ……… of rice.

e The mini-bus needs 45 ………….. of diesel.

5.10 Mensuration

Mensuration is the mathematical system of rules for finding areas and volumes.

You can use formulae to calculate area and volume.

If you choose the correct formula, you will get the correct answer!

Area is 2-dimensional.
Volume is 3-dimensional.

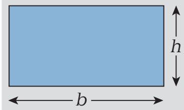
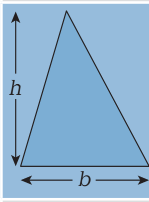
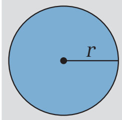
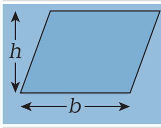
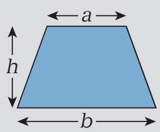
- Length:
  - use mm, cm, m, km.
  - There is only one length measurement
- Area:
  - use mm^2, cm^2, m^2, km^2.
  - Area formulae need **2** length measurements multiplied together
- Volume:
  - use mm^3, cm^3, m^3, km^3.
  - Volume formulae need **3** length measurements multiplied together
  - Volumes are also measured in millilitres and litres. $1\text{ ml} \equiv 1\text{ cm}^3$

$1\text{ litre} \equiv 1000\text{ cm}^3$

Area Formulae

Look at the diagrams – you will see **two** lengths marked on each one.

In polygons the two measurements are always at 90°. Circles are different.

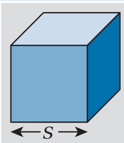
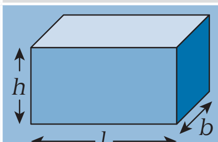
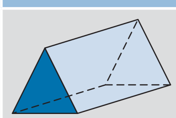
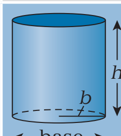
	Formula	Diagram
The area of a rectangle	$A = bh$	
The area of a triangle	$A = \frac{1}{2}bh$	
The area of a circle The radius is multiplied by itself	$A = \pi r^2$	
The area of a parallelogram	$A = bh$	
The area of a trapezium	$A = \frac{(a + b)}{2}h$	

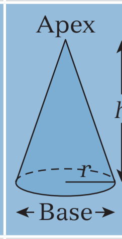
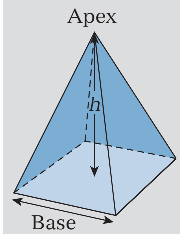
Remember:

$r^2 = r \times r$

Volume Formulae

Look at the diagrams. You will see three **lengths** marked on each solid, unless there is a circular base.

	Formula	Diagram
The volume of a cube	$V = s^3$	
The volume of a cuboid	$V = lbh$ because volume = length × breadth × height	
The volume of a prism	$V =$ Area of cross section × length	
The volume of a cylinder A cylinder is also a prism	$V = \pi r^2 h$	

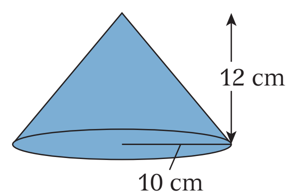
	Formula	**Diagram**
The volume of a cone	$V = \frac{1}{3}\pi r^2 h$	
The volume of a pyramid The formula is like the one for the cone	$V = \frac{1}{3} \times$ area of base $\times h$	

To calculate areas and volumes:

1. Look at the shape, or read the question carefully.
2. Do you need to find an area or a volume?
3. Find the formula for the shape you are using. Copy the formula.
4. Put the numbers from the diagram or question into the formula.
5. Calculate the value.
6. Check the units.

Example

The base has a radius of 10 cm, the height is 12 cm.
Find the volume.



1. What do you need to find? You need to find the volume.
2. What shape is it? This shape is a cone.
3. What is the formula? The formula is $V = \frac{1}{3}\pi r^2 h$
4. $V = \frac{1}{3} \times \pi \times 10 \times 10 \times 12$
5. $V = 400\,\pi$
6. $\underline{V = 1257\,\text{cm}^3}$

Exercise

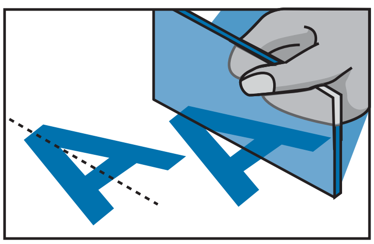
8 **Complete the table.**
The formulae, the answers and the units are given in this box:

The formulae are: $A = s^2$, $A = \frac{(a+b)}{2}h$, $V = \pi r^2 h$, $A = bh$, $C = 2\pi r$, $V = \textit{base area} \times \textit{height}$
The answers are: 49, 4.5, 503, 195, 405, 39
The units are: cm^2, m^3, mm^2, m^3, cm, cm^2

Description	Formula	Answer	Units
a A bicycle wheel has a diameter of 62 cm. How far does it travel in one turn?			
b The parallel sides of a trapezium are 5 cm and 8 cm. The distance between the parallel sides is 6 cm. What is the area of the trapezium?			
c A triangular prism has a length of 3 m. The base of the triangle measures 2 m and the height is 1.5 m. What is the volume?			
d Each square on a chessboard measures 7 cm. What is the area of each square?			
e A cylindrical storage tank has a height of 10 m. The radius of the base is 4 m. Find the volume.			
f If the base of a parallelogram measures 27 mm and the height is 15 mm, what is its area?			

5.11 Symmetry

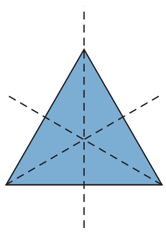
Line Symmetry



A shape with line symmetry can be folded in half so that the two halves match exactly.

The line of symmetry is like a mirror that reflects one side to match the other.

Shapes may have more than one line of symmetry.



KEY WORDS

Mirror image an image which is like a reflection in a mirror. Everything is the same, except reversed

Mirror line a line dividing a diagram or picture exactly in half

Rotational symmetry

A shape has rotational symmetry if it repeats itself more than once in every full turn (360°).

This shape has a rotational symmetry of order 4. If you turn it, it will look exactly the same every 90°.

It is possible to have line symmetry **and** rotational symmetry.

Exercise

9 **Complete this table, which lists the symmetry of the letters of the alphabet:**

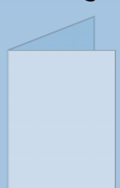
a One line of symmetry	A, T, … ?
b Two lines of symmetry	H, … ?
c Rotational symmetry of order 2	N ? ? ?
d Rotational symmetry of order 4	H, … ?
e More than 4 lines of symmetry	… ?
f No lines of symmetry, no rotational symmetry	F, G, …?
g Two lines of symmetry, rotational symmetry of order 2	… ?

Talking points

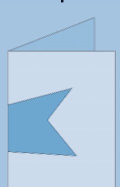
Work with a partner.

a Take a piece of paper and fold it in half.

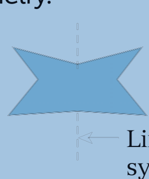
Draw a shape on the folded paper, starting and finishing at the fold line.



Cut out your shape and open it out.



Your shape has one line of symmetry.



Line of symmetry

b Make other shapes with one line of symmetry.

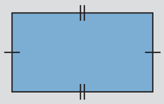
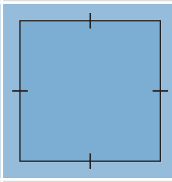
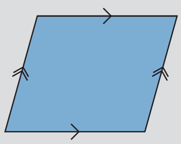
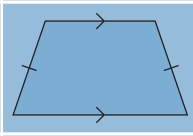
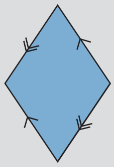
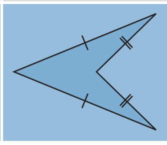
c By folding your paper twice, make shapes with two lines of symmetry.

d How can you make shapes with four lines of symmetry?

Exercise

10 **Write the name of each quadrilateral shown in this table. On a copy of the shapes, draw the lines of symmetry and write the order of rotational symmetry.**

Quadrilaterals may have line symmetry, rotational symmetry or both.

		Name	Number of lines of symmetry	Order of rotational symmetry
a			2	2
b				
c				
d				
e				
f		Arrowhead		

Think about the questions from the start of this chapter.
Can you answer them now?

- What is the sum of the angles in a triangle?
- How many sides are there in a regular heptagon?
- What is the name of the line from the centre of a circle to the edge?
- How many millimetres are there in a metre?
- What is the formula for the volume of a cone?
- How many lines of symmetry are there in a square?

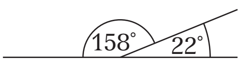
5 Consolidation: Geometry

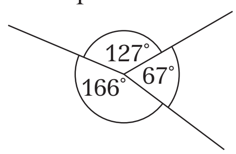
Need more practice?
Review and check your understanding here

Exercise 5.1

1 Look at the diagrams below to remind you that:

- The angles on a straight line add up to 180°
- The angles round a point add up to 360°

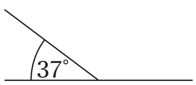




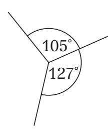
158° + 22° = 180° 127° + 67° + 166° = 360°

Use the value of the given angles to calculate the size of every other angle.

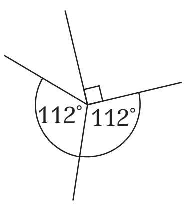
a



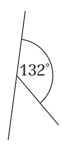
b



c

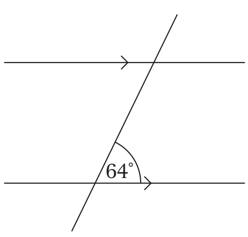


d

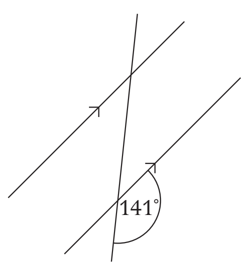


2 Use the value of the given angles to calculate the size of every other angle.

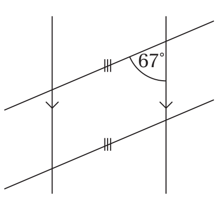
a



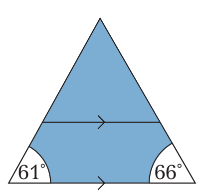
b



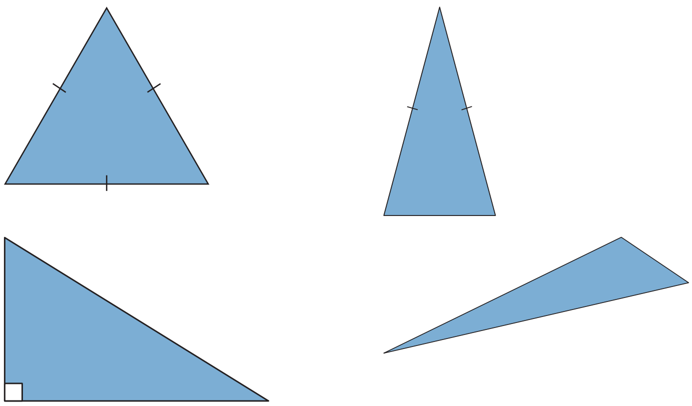
c



d



3 These diagrams show the four different types of triangle.



Look at the diagrams and then choose the correct word to complete the following sentences.

Scalene, length, isosceles, right-angled, three, two, 180°, equilateral, different, angles

You may need to use a word more than once.

a A/n ………………………triangle has ……………. angles of 60°. The sides of the triangle are all the same………………

b A/n …………………. triangle has two sides that are the same…………….and ………..angles that are the same size.

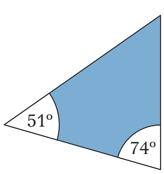
c A triangle with an angle of 90° is called a/n ……………………… triangle.

d In a/an …………………. triangle all the sides are …………….. lengths and none of the ………….. are the same.

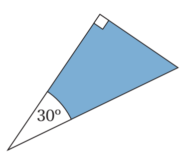
e The angles of a triangle add up to ……………

4 Use the information from question 3 to find the size of each unmarked angle in these diagrams.

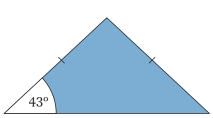
a



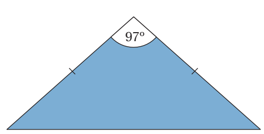
b



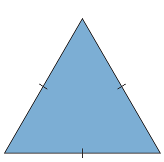
c



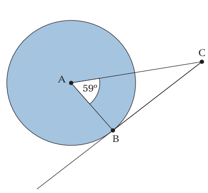
d



e



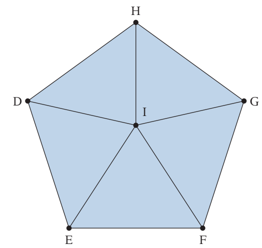
f



5 Five lines are drawn between the vertices of a regular pentagon and its centre. The pentagon is divided into five congruent triangles.

Congruent triangles are exactly the same shape and size.

If the length of the line HI is the same as the length of the line GI:



a What other lines are the same length?

b What type of triangle is drawn?

c What is the size of the angles at the centre of the pentagon?

Example

the angles DIE, EIF, …

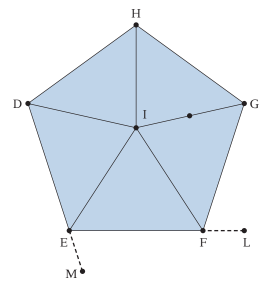
d What is the size of each internal angle?

Example

the angles DEF, EFG, …?

The lines DE and EF are extended to give external angles.

e What is the size of each external angle?

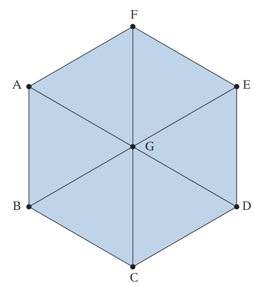


Example

MEF, LFG……

f What do the external angles add up to?

6 Look at the diagram of a regular hexagon.



a Can you give the name of the triangle AGF?

b What do you notice about the length of the sides of the triangle CGD?

c Can you calculate all the angles of triangle AGF?

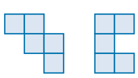
d What are the external angles of the hexagon?

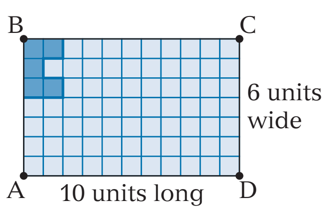
e What do the external angles add up to?

f Is this true for all regular polygons?

Extension

7





These are pentominos, shapes created by linking five squares together using the edges of the squares.

There are twelve different shapes altogether. Can you find them? Be careful about rotations or reflections.

When you have found all twelve shapes, you can fit all the pentominos into a rectangle, 10 units long and × 6 units wide. It is easier to do this if you cut them out, rather than draw them!

Exercise 5.2

You will need a calculator for this exercise. Give your answers to 3 significant figures.

1 The radius of a circle is 12 cm. Find its circumference and its area.

2 The diameter of a circle is 8m. Find its circumference and its area.

3 A circle has a circumference of 44 cm.

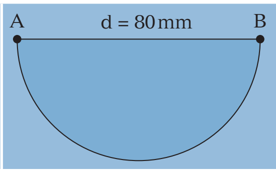
a What is the radius of the circle?

b What is the diameter of the circle?

4 The distance from the centre of a circle to its circumference is 2.5 m. Calculate the area of the circle.

5

A circle cut in into two equal pieces is called a semicircle. What is the perimeter of this semicircle?



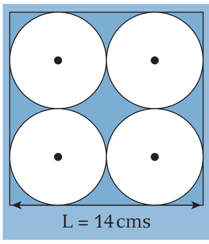
6

Look at this diagram.

a What is the area of the square?

b What is the total area of the four circles?

c What is the area of the shaded part inside the square?



Reading Practice!

Read these questions carefully, then:

- Identify the shape
- Draw the shape and mark the measurements
- Find the correct formula
- Use this formula to calculate the area or volume.

7 Find the area of a parallelogram with a base that measures twelve centimetres and a height that measures seven centimetres.

8 A right-angled triangle has a base measuring two and a half metres, and a height of one and a half metres. Calculate the area.

9 Calculate the volume of a cylinder. The height of the cylinder is twelve millimetres and the radius of the base is five millimetres.

10 A trapezium has parallel sides that measure seven centimetres and nine centimetres. The distance between the parallel sides is five centimetres. Work out the area of the trapezium.

11 A box has a length of fourteen centimetres, a width of ten centimetres, and a height of eight centimetres. What is the volume of the box?

12 What is the volume of a pyramid with a square base of eight centimetres and a height of five and a half centimetres?

13 Find the area of a carpet that is three and a quarter metres long, and two and one third metres wide.

Similar shapes

14 **a** Calculate the area of a square with sides of 3 cm.

b Calculate the area of a square with sides that are 9 cm long.

c Write the ratio of their areas in its simplest form.

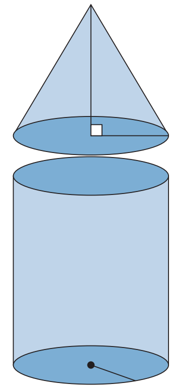
15 **a** Calculate the volume of a cube with sides of 4 cm.

b Calculate the volume of a cube with sides of 8 cm.

c Write the ratio of the volumes in its simplest form.

16 Fill in this table. Some of the entries have been made for you.

Ratio of length	Ratio of area	Ratio of volume
1 : 2	1 : 4	
1 : 3		1 : 27
1 : 5		
1 : a	1: a^2	



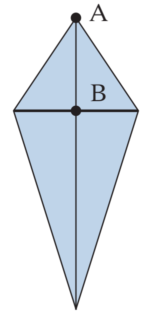
Practical mensuration

17 Matt is building a rocket. The rocket is made from a cone and a cylinder.

The radius of the cone and cylinder is 3.5 cm and the height of the cylinder is 30 cm.

The length of the rocket is 40 cm.

Find the volume of the rocket.



18 Jaime is making a kite. She uses two sticks and some thick paper.

One stick is 30 cm long and the other is 50 cm long.

Label this diagram with the lengths. AB is 14 cm.

What is the area of the paper used in the kite?

Extension

The Platonic solids are the five solid shapes that were studied by the mathematicians in Greece, more than two thousand years ago.

The flat 2-D (two dimensional) diagrams are called 'nets' and if you copy them onto thick paper and then fold them, you can make the solid shapes.

Can you find out the name of each solid?

Name of solid	3-D image	Net